International Journal of Theoretical Physics, Vol. 43, No. 3, March 2004 (© 2004)

Gravity-Mediated Modifications of the Dispersion Relation in Nontrivial Backgrounds

Daniel Arteaga,[1,4] Renaud Parentani,[2] and Enric Verdaguer[3]

Radiative corrections evaluated in nontrivial backgrounds lead to dispersion relations which effectively break the local Lorentz symmetry even if Lorentz invariance holds at a fundamental level. We report on progress made toward the calculation of radiative corrections which are induced by gravity. These should be relevant when approaching Planck scale. We first present the properties of the self-energy of a scalar particle traveling in a thermal graviton bath. We then discuss the possibility of performing the corresponding calculation in a curved background. We give the connection between two different approaches to the dispersion relation, based on the self-energy and the effective action, and we emphasize the need for the closed-time-path formalism in curved backgrounds.

KEY WORDS: Lorentz symmetry breaking; quantum gravity; effective field theories.

1. INTRODUCTION

The possibility of Lorentz-breaking-dispersion relations has recently received much attention. On the one hand, the possible observation of ultrahigh energetic cosmic rays beyond the Greisen–Zatsepin–Kuzmin (GZK) cutoff (Takeda *et al.*, 1998) has lead some authors to speculate about the modified dispersion relations in order to account for this observation (Amelino-Camelia and Piran, 2001a,b; Kifune, 1999). These modified dispersion relations have been mainly motivated by several approaches to quantum gravity and string theory, which suggest that Lorentz symmetry could be broken at a fundamental level (Alfaro *et al.*, 2000, 2002; Corroll *et al.*, 2001; Kostelecky and Samuel, 1989). Several authors (Carroll

[1] Departament de Física Fonamental, Universitat de Barcelona, Barcelona, Spain.
[2] Laboratoire de Physique Théorique, CNRS UMR 8627, Université Paris XI, 91405 Orsay Cedex, France.
[3] Departament de Física Fonamental and CER en Astrofísica, Cosmologia i Física de partícules, Universitat de Barcelona, Barcelona, Spain.
[4] To whom correspondence should be addressed at Departament de Física Fonamental, Universitat de Barcelona, Av. Diagonal 647, 08028 Barcelona, Spain; e-mail: darteaga@ub.edu.

et al., 1990; Jacobson *et al.*, 2003a,b, 2004; Kostelecký and Mewes, 2001) have studied the limits on the possible amount of Lorentz violation from the current astrophysical observational data.

On the other hand, modified dispersion relations have also been considered in the context of the *trans-Planckian problem* which appears both in black hole physics and in inflationary cosmology. In black hole physics, the quanta responsible for Hawking radiation at late times correspond to vacuum fluctuations of arbitrarily high energy near the black hole horizon (Jacobson, 1991, 1993, 1999). Similarly in inflationary cosmology, modes at the origin of the large-scale structures had length scales much smaller than the Planck length in the early stages of inflation. It is therefore of interest to determine to what extent the properties of Hawking radiation (Brout *et al.*, 1995; Corley and Jacobson, 1996; Helfer, 2003; Unruh, 1995) and those of the primordial fluctuation spectrum (Martin and Brandenberger, 2001, 2003; Niemeyer, 2001; Niemeyer and Parentani, 2001) are sensitive to modifications of the dispersion relation at the Planck scale.

Many approaches to the issue the modification of the dispersion relation rely on the assumption that Lorentz symmetry could be broken at a fundamental level. However, even if the Lorentz group is taken to be a symmetry of the underlying fundamental theory, quantum gravity effects might introduce nontrivial dispersion relations in black hole physics and in cosmology. A dynamical realization of this line of thought was pursued in Parentani (2001, 2002), following 't Hooft (1996) observation that strong gravitational interactions in the near horizon region might alter the semiclassical description of black hole evaporation.

In fact it is well known that quantum effects evaluated in nontrival backgrounds may induce an effective breaking of the local Lorentz invariance.[5] In the case of QED, Adler (1971) already recognized that photons propagating under the presence of a strong magnetic field would propagate at speeds smaller than c, thereby effectively breaking the local Lorentz symmetry.

Drummond and Hathrell (1980) realized that electromagnetic quantum corrections in curved spacetimes would also alter the characteristics of propagation of photons. They computed the modification to the speed of light of low-energy photons propagating in curved space-times, and found the surprising result that in many physical situations light would travel at speeds greater than c. Shore (2002a,b) generalized the Drummond and Hathrell result in order to include high-energy photons and to account for dispersion. He found that the prediction of superluminal photon velocity was apparently exact to all frequencies. We refer

[5] To avoid confusion, let us emphasize that we do not claim tht the Lorentz symmetry of the full theory is broken. What we mean is that radiative corrections to self-energies evaluated in thermal heat baths or in curved backgrounds contain terms which depend on quantities such as vector fields or tensors, and not only on the metric evaluated in the tangent plane, as it is the case in the vacuum in Minkowski space-time.

to Shore (2003) for a review of this topic and a discussion of the implications concerning causality.

Similarly, in the presence of a thermal QED heat bath the effective speed of the photons is lowered (Tarrach, 1983) and the fermion dispersion relation is modified (Donoghue *et al.*, 1985). Finally we can also mention that Scharnhorst (1990) and Barton (1990) worked out the propagation of light between two Casimir plates and found that the speed of light was increased.

Most calculations considering effective modifications of the dispersion relation involve the electromagnetic interaction. However, at energies approaching the Planck scale (and hence relevant for the situations concerning the trans-Planckian problem) gravitational interaction should dominate. Since our primary concern is the trans-Planckian problem it is worth studying the gravity-mediated corrections of self-energies. Additionally note that gravity is universal and not limited to charged particles. Although matter coupled to gravity is a nonrenormalizable theory, low-energy predictions which do not depend on the Planck scale behavior of gravity can be extracted in the spirit of effective field theories (Donoghue, 1994a,b; Weinberg, 1995).

A first step to study gravity-induced modifications of the dispersion relation was undertaken in a recent paper (Arteaga *et al.*, 2004). In that reference the Lorentz-breaking modifications of the dispersion relation of a scalar particle interacting with a thermal graviton bath were extracted from the poles of the retarded propagator. However, since this procedure requires the notion of momentum space, it is not clear how to generalize it to curved spacetimes. In fact Drummond and Hathrell (1980) used a different approach: they computed the effective QED action in a curved spacetime, and from that they derived the effective equations of motion for the photon. The dispersion relation was then recovered through a geometric optics approximation.

In this paper we further consider modified dispersion relations arising from gravitational interactions. We discuss the possibility of computing the corrections in a curved spacetime. In particular, we point out the connection between the self-energy and the effective action, and compare the dispersion relations obtained from the effective action and from the self-energy, stressing the importance of a closed-time-path (CTP) formalism in curved spacetime.

For completeness let us mention that, working in the context of brane world scenarios, Burgess *et al.* (2002) considered the gravity-mediated modifications of the dispersion relation of brane-bound fermions and photons. In their case the primary source of Lorentz-violation were some extradimensional configurations. Let us also mention that Borgman and Ford (2003) considered the fluctuations of a bundle of geodesics in a flat thermal spacetime, considering the backreaction of the scalar field on the metric perturbations. However their approach was semiclassical. To consistently incorporate backreaction effects at the leading order in our quantum theoretical framework we should work in the large N limit and consider the dressed graviton propagator obtained either from field theory (Tomboulis, 1977) or from stochastic gravity (Hu and Verdaguer, 2003, 2004; Martín and Verdaguer, 2000).

The plan of the paper is the following. In Section 2 the propagation of a scalar field in a graviton background is studied, summarizing the main results of Arteaga *et al*. (2004). In Section 3 we report on some work in progress concerning the propagation in generally curved backgrounds. Finally, in Section 4 we analyze the connection between the self-energy and the effective action methods for the derivation and study of the modified dispersion relations.

We use a system of units with $\hbar = c = k_{\rm B} = 1$. The signature of the metric is $(-, +, +, +)$.

2. THERMAL FLAT BACKGROUND

In this section we consider the propagation of a scalar particle in a thermal bath. Our aim is to determine how gravitational radiative corrections modify the dispersion relation of the particle. For this we shall compute the corrections to the self-energy.

2.1. The System

Let us consider a minimally coupled real scalar field ϕ of mass m which propagates in a spacetime characterized by a metric $g_{\mu\nu}$. The action for the field is

$$S_{\phi,g} = -\int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2}m^2\phi^2\right), \tag{1a}$$

and the action for the metric is

$$S_g = \frac{2}{\kappa^2}\int d^4x\sqrt{-g}R. \tag{1b}$$

where $\kappa = \sqrt{32\pi G} = \sqrt{32\pi}L_{\rm Pl}$ is the gravitational coupling constant. Assuming that the metric is a small perturbation of Minkowski spacetime, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, the action $S = S_{\phi,g} + S_g$ can be decomposed into the free scalar field, graviton, and interaction actions, $S = S_\phi + S_h + S_{\rm int}$, as

$$S_\phi = \int d^4x\left(-\frac{1}{2}\,\partial_\mu\,\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2\right), \tag{2a}$$

$$\begin{aligned} S_h = \int d^4x\bigg(&-\frac{1}{2}\,\partial^\alpha h^{\mu\nu}\,\partial_\alpha h_{\mu\nu} + \partial_\nu h^{\mu\nu}\,\partial^\alpha h_{\mu\alpha} - \partial_\mu h\,\partial_\nu h^{\mu\nu}\\ &+\frac{1}{2}\,\partial^\mu h\,\partial_\mu h\bigg) + O(\kappa), \end{aligned} \tag{2b}$$

$$S_{\text{int}} = \frac{\kappa}{2}\int d^4x T^{\mu\nu} h_{\mu\nu} + O(\kappa^2), \tag{2c}$$

where $h = h^{\mu}_{\mu}$ is the trace of the perturbation and

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\,\partial_\alpha\phi\,\partial^\alpha\phi - \frac{1}{2}\eta_{\mu\nu}m^2\phi^2 \tag{3}$$

is the stress tensor of the scalar field. Indices are raised and lowered with the background metric $\eta_{\mu\nu}$.

Since gravitational interactions are non-renormalizable, the system of the scalar field coupled to gravity should be conceived as an effective field theory (Donoghue, 1994a,b; Weinberg, 1995). To compute to a given precision $\mathrm{E}^n\kappa^n$, where E is the energy of the process, one has to introduce all possible counterterms compatible with the symmetry whose coefficients are of order κ^n at most. In practice this implies that to order κ^2, additionally to the well-known mass and field strength counterterms, one has to add a four-derivative counterterm:

$$S_{\text{count}} = -\int d^4x\left[\frac{1}{2}\left(m_0^2 - m^2\right)\phi^2 + \frac{1}{2}(Z-1)\,(\partial_\mu\phi\;\partial^\mu\phi + m^2\phi^2)\right.$$
$$\left. + \frac{1}{4}\kappa^2 C_o\,(\partial_\mu\,\partial^\mu\phi)^2\right] + O(\kappa^4). \tag{4}$$

The coefficients m_0, Z, and C_0 correspond to unobservable bare quantities. Additionally the graviton action Eq. (2b) must be supplemented with a gauge-fixing term. No Fadeev–Popov ghost fields are needed since we do not consider graviton self-interaction.

2.2. Zero Temperature Self-Energy

At zero temperature the self-energy $\Sigma^{(T=0)}(p^2)$ is defined through

$$G_{\mathrm{F}}^{(T=0)}(p) = \frac{-i}{p^2 + m^2 + \Sigma^{(T=0)}(p^2)}, \tag{5}$$

where $G_{\mathrm{F}}^{(T=0)}(p)$ is the zero temperature Feynman propagator of the scalar field. We recall that the self-energy can be computed as the sum of all one-particle irreducible diagrams with amputated external legs (Peskin and Schroeder, 1998; Weinberg, 1995). The two diagrams which may contribute to order κ^2 are shown in Fig. 1. Additionally, one must also take into account the contribution of the counterterms:

$$\Sigma^{(T=0)}(p^2) = \left(m_0^2 - m^2\right) + (Z-1)(p^2 + m^2) + \kappa^2 C_0 p^4$$
$$+ \Sigma_1^{(T=0)}(p^2) + \Sigma_2^{(T=0)}(p^2) + O(\kappa^4). \tag{6}$$

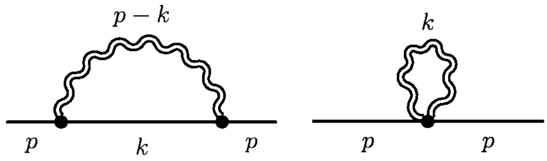

Fig. 1. The two Feynman diagrams needed for the calculation of the self-energy, respectively $\Sigma_{(1)}(p)$ and $\Sigma_{(2)}(p)$.

The relevant contribution comes from the first diagram, since the second diagram is a massless tadpole, which automatically vanishes if we empoly dimensional regularization. Once we include the contribution of the counterterms the renormalized self-energy is found to be (for the details of the calculation see Arteaga *et al.*, 2004):

$$\begin{aligned}\Sigma^{(T=0)}(p^2) = & -\frac{\kappa^2}{(4\pi)^2}\left(\frac{m^6}{2p^2}+\frac{m^4}{2}\right)\ln\left(1+\frac{p^2}{m^2}-i\epsilon\right)\\ & -\frac{\kappa^2}{(4\pi)^2}(m^4+m^2p^2)\ln\left(\frac{p^2+m^2}{\mu^2}-i\epsilon\right)\\ & +C\kappa^2(p^2+m^2)^2+O(\kappa^4).\end{aligned} \tag{7}$$

The finite coefficient C corresponds to the unknown finite part of the four-derivative counterterm. In principle its value should be determined by experiments or by the knowledge of the underlying more fundamental theory. However in this particular case there are no divergences associated to the p^4 term (see Arteaga *et al.*, 2004), so that it would have been consistent not to include the four-derivative counterterm and simply take $C = C_0 = 0$ from the beginning.

2.3. Finite Temperature Self-Energy

We incorporate the thermal effects into the self-energy through the real-time description of thermal field theory (Das, 1997; Landsman and van Weert, 1987; le Bellac, 1996), which can be seen as a particular application of the CTP method in nonequilibrium field theory (Chou *et al.*, 1985; Keldysh, 1965; Schwinger, 1961).

In the CTP approach, which can be applied for an arbitrary initial state $\hat{\rho}$, the number of degrees of freedom is doubled. One has to consider four propagators organized in a 2×2 matrix $G_{ab}(x, x')$. The 11 and 22 components correspond respectively to the Feynman (time-ordered) and Dyson (anti-time-ordered) propagators, and the off-diagonal components correspond to the Wightman functions (nonordered). The self-energy also becomes a matrix $\Sigma^{ab}(x, x')$, defined through

$$G_{ab}(x, x') = G^{(0)}_{ab}(x, x') + \int d^4y\, d^4z G^{(0)}_{ac}(x, y)[-i\Sigma^{cd}(y, z)]G_{db}(z, x') \tag{8}$$

where $G^{(0)}_{ab}(x, x')$ are the propagators of the free theory. Perturbation theory can be organized as usual, but taking into account that there will be two kinds of vertices and four kinds of propagators. Fourier-tranformed propagators can be introduced as follows (Chou *et al.*, 1985):

$$\tilde{G}_{ab}\,(p; X) = \int d^4\Delta\; e^{-ip\cdot\Delta}\; G_{ab}(X + \Delta/2, X - \Delta/2), \tag{9}$$

where we have introduced the new variables $X^\mu = (x^\mu + x'^\mu)/2$ and $\Delta^\mu = x^\mu - x'^\mu$. An analogous definition applies for the momentum representation of the self-energy, $\Sigma^{ab}\,(p; X)$. To simplify the notation we will drop the tilde when referring to Fourier-transformed quantities.

An interesting combination is the retarded propagator $G_{\rm R}(p; X) = G_{11}(p; X) - G_{12}(p; X)$: it exhibits simple analytical properties (analyticity in the upper p^0-plane), it is directly connected with the retarded self-energy $\Sigma_{\rm R}\,(p; X) = \Sigma_{12}(p; X) + \Sigma_{11}(p; X)$ through

$$G_{\rm R}\,(p; X) = \frac{-i}{p^2 + m^2 + \Sigma_{\rm R}\,(p; X)}, \tag{10}$$

and, furthermore, at finite temperature the position locations of its poles have well-defined interpretations in terms of energies and thermalization rates. It is work emphasizing that Eq. (10) holds in a generic nonequilibrium situation, while the more familiar relation Eq. (5) is specific of zero temperature field theory.

In the thermal case the initial density matrix is taken to be $\hat{\rho} = e^{-\beta\hat{H}}/\mathrm{Tr}\,(e^{-\beta\hat{H}})$, and propagators do no longer depend on X in the momentum representation, since the state is homogeneous. The free propagators $G^{(0)}_{ab}\,(p)$ contain the usual vacuum contribution plus a thermal on-shell part proportional to the Bose–Einstein distribution function,

$$n(E) = \frac{1}{e^{\beta E} - 1}. \tag{11}$$

This last term accounts for the presence of real particles on the thermal bath and breaks Lorentz invariance through the dependence in E, the zero component of p_μ evaluated in the rest frame of the heat bath. For more details we refer to the aforementioned references and to Appendix C of Arteaga *et al.* (2004).

We study separately the real and imaginary parts of the retarded self-energy. To compute the real part we have to evaluate both diagrams $\Sigma_{(1)}(p)$ and $\Sigma_{(2)}(p)$, shown in Fig. 1. In Arteaga *et al.* (2004) we show that at low temperatures the real part of the self-energy it is given by

$$Re\Sigma_{\rm R}\,(E_{\mathbf{p}}, \mathbf{p}) = -\frac{1}{6}\kappa^2 m^2 T^2 - \kappa^2\sqrt{\frac{m^5T^3}{8\pi^3}}\left(\frac{m^2 + 2|\mathbf{p}|^2}{3m^2 + 4|\mathbf{p}|^2}\right) e^{-m/T}, \quad T \ll m. \tag{12}$$

We have directly quoted the on-shell result ($p^0 = E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$) since it is the one we shall need afterwards for the dispersion relation. The first term on the right-hand side of Eq. (12) is a constant contribution which comes from the thermal real gravitons in the background. It is somewhat a surprise that this term does not depend on the 3-momentum since the original integrals were not explicitly Lorentz-invariant. It must be noted that a completely analogous situation happens in electrodynamics (Donoghue *et al.*, 1985). The second term on the right-hand side of Eq. (12), which depends on the particle 3-momentum, comes from the thermal scalar particles in the bath. Since there are almost no massive particles at low temperatures, this second contribution is exponentially suppressed. This damping disappears at high temperature, where there are many scalar particles in the bath. In this regime one obtains

$$Re\Sigma_{\mathrm{R}}\left(E_{\mathbf{p}}, \mathbf{p}\right) = \frac{1}{48}\kappa^2 m^2 T^2 \left[-11 + \frac{E_{\mathbf{p}}}{|\mathbf{p}|}\ln\left(\frac{2E_{\mathbf{p}} + |\mathbf{p}|}{2E_{\mathbf{p}} - |\mathbf{p}|}\right)\right], \quad T \gg m. \tag{13}$$

In principle the imaginary part of the self-energy could be computed in a similar way, or, alternatively, with the aid of the cutting rules at finite temperature. In both cases, one finds a finite nonzero result. However, as we shall see in the next subsection, to order κ^2 the imaginary part of the self-energy vanishes when evaluated on-shell. It can be shown that the nonzero result comes from the infrared divergence of the Bose–Einstein thermal factor. Once this infrared behavior is regulated, for instance by giving a tiny mass to the graviton, no imaginary part is found.

2.4. Dispersion Relation

At zero temperature, the position of the pole of the propagator gives the energy of the state and hence defines the dispersion relation, according to the Källen–Lehmann spectral representation (Peskin and Schroeder, 1998; Weinberg, 1995). In the present case it is found to be

$$(p^0)^2 = m^2 + |\mathbf{p}| + \Sigma^{(T=0)}\left(-m^2\right) = m^2 + |\mathbf{p}|^2, \tag{14}$$

where the second equality is a consequence of the on-shell renormalization scheme (the renormalized mass m coincides with the physical mass of the particle). The dispersion relation is clearly Lorentz-invariant, as expected.

At finite temperature, Donoghue *et al.* (1985) showed that the "effective" dispersion relation of the particle, given by the real part of the poles of the retarded propagator, determines the intertial properties of the particle (see also Section 4 in this paper). The real part of the poles is given by

$$(p^0)^2 - |\mathbf{p}|^2 = m^2 + \mathrm{Re}\,\Sigma_{\mathrm{R}}(p^0, \mathbf{p}). \tag{15}$$

The thermal mass is obtained by setting $\mathbf{p} = \mathbf{0}$:

$$m_T^2 = m^2 + \mathrm{Re}\Sigma_{\mathrm{R}}\,(m_T, \mathbf{0}). \tag{16}$$

In a Lorentz-invariant situation one would simply have $(p^0)^2 = m_T^2 + |\mathbf{p}|^2$, but in general there can be additional dependence on the 3-momentum $\mathbf{p}$ on the righthand side,

$$(p^0)^2 = m_T^2 + |\mathbf{p}|^2 + \mathcal{F}(\kappa, T, m_T, \mathbf{p}). \tag{17}$$

The Lorentz-breaking term in the dispersion relation leads to modifications of the group velocity of the particles $v = dp^0/dp$ (Latorre *et al.*, 1995):

$$v = \frac{\mathbf{p}}{p^0} + \frac{1}{2p^0}\frac{\partial \mathcal{F}}{\partial \mathbf{p}}. \tag{18}$$

As shown in Section 4, if the initial state were an arbitrary non-equilibrium state instead of a thermal one, the same analysis would apply, but in this case there would be an explicit dependence on the position X in the self-energy.

Let us find the explicit form of the thermal mass and Lorentz-breaking terms both in the low- and high-temperature regimes. Equation (15) can be solved perturbatively:

$$(p^0)^2 = m^2 + |\mathbf{p}|^2 + Re\ \Sigma_{\mathrm{R}}(E_{\mathbf{p}}, \mathbf{p}) + O(\kappa^4), \tag{19}$$

where we recall that $E_{\mathbf{p}} = \sqrt{m^2 + |\mathbf{p}|^2}$. At low temperatures the modified dispersion relation, according to Eqs. (12) and (19), is approximately given by

$$(p^0)^2 = m_T^2 + |\mathbf{p}|^2 - \kappa^2\sqrt{\frac{m^5T^3}{2\pi^3}}\left(\frac{|\mathbf{p}|^2}{3m^2 + 4|\mathbf{p}|^2}\right)e^{-m/T}, \tag{20}$$

where the leading contribution to the thermal mass is

$$m_T^2 = m^2 - \frac{1}{6}\kappa^2 m^2 T^2. \tag{21}$$

Notice that at low temperatures the Lorentz-breaking term carries a Boltzmann factor $e^{-m/T}$. This is again due to fact that the non-trival momentum dependence comes from the thermal scalar particles whose abundance is exponentially suppressed at low temperatures. Analogously to what happens in electrodynamics (Donoghue *et al.*, 1985), the effect of the graviton bath only shows up in the thermal mass.

At high temperature, $T \gg m$, the modified dispersion relation, according to Eqs. (13) and (19), is

$$(p^0)^2 = m_T^2 + |\mathbf{p}|^2 + \frac{\kappa^2 m^2 T^2}{48}\left[\frac{E_{\mathbf{p}}}{|\mathbf{p}|}\ln\left(\frac{2E_{\mathbf{p}} + |\mathbf{p}|}{2E_{\mathbf{p}} - |\mathbf{p}|}\right) - 1\right], \tag{22}$$

and the thermal mass is

$$m_T^2 = m^2 - \frac{11}{48}\kappa^2 m^2 T^2. \tag{23}$$

The group velocity is given by

$$\mathbf{v} \approx \frac{\mathbf{p}}{p^0}\left[1 + \frac{1}{96}\frac{\kappa^2 T^2 m^4}{(4m^2 + 3|\mathbf{p}|^2)|p|^2} - \frac{1}{96}\frac{\kappa^2 T^2 m^4}{|\mathbf{p}|^3 E_{\mathbf{p}}}\ln\left(\frac{2E_{\mathbf{p}} + |\mathbf{p}|}{2E_{\mathbf{p}} - |\mathbf{p}|}\right)\right], \quad T \gg m. \tag{24}$$

At high temperature the terms which break the Lorentz symmetry are no longer exponentially suppressed. Notice also that Eqs. (22) and (24) show that there is no modification to the dispersion relation for massless scalars. Ultrarelativistic massive particles do not show either significative deviations from the Lorentz-symmetric dispersion relation.

While the real part of the self-energy gives the change in energy of the particle and hence the dispersion relation, the imaginary part accounts for the dissipative effects and gives the thermalization rate. In a generalization of the optical theorem to finite temperature, the imaginary part of the self energy can be expressed as the sum of all processes which contribute to the decay rate minus the sum of all processes which contribute to the creation rate (Weldon, 1983). But to order κ^2 a real particle can neither emit nor absorb a real graviton, because there is no phase space available for those processes. Hence, at this order, the imaginary part of the self-energy must vanish on shell. To account for thermalization effects we should compute the imaginary part of the self-energy to order κ^4. At this order, the particle can exchange momentum with the thermal bath through processes analogous to Compton and Coulomb scattering in electrodynamics.

3. CURVED BACKGROUNDS

It is not immediate how to compute the dispersion relation of a particle in a curved background following a method similar to the one described in the previous section. Although the real-time approach to field theory may be understood as an application of the CTP formalism, and this formalism is well suited for curved spacetimes, the above procedure heavily relies on the the momentum representation of the propagator, which is not defined in general in curved spacetimes. Therefore the notion of dispersion relation is ambiguous in curved spacetimes.

However, when the length scales associated with the particle propagation are much smaller than the characteristic curvature radius of the spacetime L, it should be possible to approximately recover the notion of dispersion relation. Bunch and Parker (1979) showed that, expanding in powers of the inverse curvature radius, it was possible to define a perturbative momentum representation for the free propagator (see also Birrell and Davies, 1982):

$$G^{(0)}(x,x') = \int \frac{d^4p}{(2\pi)^4} \exp\left(ip_\mu x^\mu_{x'}\right)\tilde{G}^{(0)}(p;x'), \tag{25a}$$

where $x^\mu_{x'}$ are the coordinates of the point x in a system of normal coordinates centered around x'. The Fourier components are

$$\tilde{G}^{(0)}(p;x') = \frac{-i}{p^2+m^2} - \frac{i\left(\frac{1}{3}-\xi\right)R(x')}{(p^2+m^2)^2} + \frac{2iR_{\alpha\beta}(x')p^\alpha p^\beta}{3(p^2+m^2)^3} + O(L^{-3}), \tag{25b}$$

with ξ being the conformal coupling factor of the field. Depending on the integration contour for p^0 (or, alternatively, on the choice of the $-i\epsilon$ terms for the singularities), $G^{(0)}$ can represent the Feynman, the retarded, or any other propagator (evaluated in the local adiabatic vacuum). The above expansion is found to be equivalent to the Schwinger-de Witt, or adiabatic, expansion of the propagator.

The normal coordinates appearing in the propagator have a natural interpretation as the locally Minkowskian coordinates at the observation point. However they limit the practical usefulness of the momentum representation as a tool for computing the finite part of the Feynman diagrams in curved spacetimes, since Feynman rules require integration over all spacetime. Moreover momentum would be no longer conserved through the diagram, as can be seen from the explicit x' dependence of Eq. (25b). Therefore it may prove more convenient to compute the propagator with some coordinate representation and then eventually use a development similar to that of Eq. (25b) to interpret the result. To compute the propagator one may try to study some particular case with full generality, or, alternatively, study a generic situation with an adiabatic or weak-field approximation. However, it remains to be seen whether an expresion analogous to Eq. (25b) will continue to hold for the dressed propagator. We can now explicitize what we mean by "radiative corrections which break the local Lorentz invariance." In the p, x' representation of the dressed propagator, these terms would be scalars in the tangent x'-plane which depend on some non-trivial tensor fields and which therefore cannot be absorbed in a redefinition of the parameters which already exist in Eq. (25b), i.e. the mass, the parameter ξ, and the normalization of residue of the pole.

Drummond and Hathrell (1980) followed a different procedure in order to study the modified dispersion relations. We recall that they considered the corrections to the photon propagation induced by electromagnetic interactions in curved spacetimes. Instead of studying the location of the poles of the propagator, they computed the photon-effective action, and from that they extracted the effective equations of motion. Then, the effective action was evaluated under a weak-field approximation for gravity following two different equivalent methods: a Schwinger-de Witt expansion and an independent diagrammatic technique. They also assumed low frequencies, but this second assumption was later removed by Shore (2002a,b). To derive the dispersion relation from the equations of motion they used a geometric optics approximation: they assumed that the solution was of the form $A(x)\ \exp[iS(x)]$, with the phase $S(x)$ varying much more rapidly than $A(x)$. For the free electromagnetic action this leads to $k^2 = 0$, with $k_\mu(x) = \nabla_\mu S(x)$. When the effective action is considered corrections to the usual dispersion relation are found.

The approach by Drummond and Hathrell has the advantage that it does neither rely on the momentum representation nor on the study of the poles of the propagator. We will see in the next section how this approach is connected to the more usual pole analysis in flat spacetime.

Additionally, it is worth noticing that Drummond and Hathrell employed the Feynman in-out formalism in their calculation. However in non-equilibrium situations the effective equations of motion deduced from the in-out effective action do not correspond in general to the equations of motion of the expectation value of the field, but rather to the in-out transition elements (Jordan, 1986). To get equations of motion for the true expectation value it is necessary to compute the CTP effective action. We will study in the next section whether using the in-out approach or the CTP formalism makes any difference in the Drummond and Hathrell's case.

4. SELF-ENERGY AND EFFECTIVE ACTION METHODS

In this section we shall compare the effective action and self-energy approaches to the dispersion relation. To this end we will study the equations of motion derived from the CTP effective action in curved spacetimes (Calzetta and Hu, 1987; Campos and Verdaguer, 1994, 1996; Chou *et al.*, 1985; Jordan, 1986). We will allow for an arbitrary initial state $\hat{\rho}$.

The CTP effective action $\Gamma[\bar{\phi}_1, \bar{\phi}_2]$ is defined from the Legendre transform of the CTP generating functional, in a similar way as the usual in-out effective action (which can be recovered by setting $\bar{\phi}_2 = 0$) but taking into account the CTP doubling of degrees of freedom. Functionally differentiating the CTP effective action we get the effective equations of motion for the expectation value of the field, $\bar{\phi} = \mathrm{Tr}(\hat{\rho}\hat{\phi})$:

$$\left.\frac{\delta\Gamma[\bar{\phi}_1, \bar{\phi}_2]}{\delta\bar{\phi}_1(x)}\right|_{\bar{\phi}_1=\bar{\phi}_2=\bar{\phi}} = 0. \tag{26}$$

These equations of motion are real and causal (Jordan, 1986) because they correspond to equations of motion of true expectation values.

The effective action can always be expanded as

$$\Gamma[\bar{\phi}_1, \bar{\phi}_2] = \sum_r \frac{1}{r!}\int \sqrt{-g(x_1)}d^4x_1 \cdots \sqrt{-g(x_r)}d^4x_r \times\Gamma^{a_1\cdots a_r}(x_1, \cdots, x_r)\bar{\phi}_{a_1}(x_1)\cdots\bar{\phi}_{a_r}(x_r), \tag{27}$$

where we have used an Einstein summation convention for repeated CTP indices $a_i \epsilon \{1, 2\}$. The coefficients $\Gamma^{a_1 \cdots a_r}(x_1, \ldots, x_r)$ are called proper vertices. A straightforward generalization of the usual argument (see e.g. Peskin and Schroeder, 1998) shows that this 2-point vertex corresponds to the inverse propagator,

$$\Gamma^{ab}(x, y) = i(G^{-1})^{ab}(x, y). \tag{28}$$

The equation which defines the self-energy $\Sigma^{ab}(x, x')$,

$$\begin{aligned} G_{ab}(x, y) = G^0_{ab}(x, y) - i \int \sqrt{-g(z)}\, d^4 z \sqrt{-g(w)}\, d^4 w\, G^0_{ac}(x, z) \\ \times \Sigma^{cd}(z, w) G_{cb}(w, y), \end{aligned} \tag{29}$$

can be manipulated to give

$$(G^{-1})^{ab}(x, y) = A^{ab}(x, y) + i \Sigma^{ab}(x, y), \tag{30}$$

where $A^{ab}(x, y)$ is the inverse of the free propagator,

$$A^{ab}(x, y) = [(G^{(0)})^{-1}]^{ab}(x, y) = c^{ab}[-g(x)]^{-1/2} i(-\Box^2 + m^2)\, \delta^{(4)}(x - y), \tag{31}$$

with $c^{ab} = \mathrm{diag}(1, -1)$. We see that the 2-point vertex can be expressed as

$$\Gamma^{ab}(x, y) = i A^{ab}(x, y) - \Sigma^{ab}(x, y). \tag{32}$$

Hence the 2-point vertex essentially corresponds to the self-energy, which can be computed as the sum of all one-particle irreducible (1PI) diagrams. Other proper vertices also have similar interpretations in terms of 1PI diagrams.

Let us suppose now that the relevant vertex is the two-particle vertex Γ^{ab}. This hypothesis can be justified in many situations: tadpole contributions Γ^a vanish with usual renormalization conditions, and if we are performing a leading order calculation, n-particle vertices, with $n > 3$, are often of higher order in the coupling constant (this is indeed the case of QED and gravity). In this case the effective equations of motion can be expressed as

$$\left. \frac{\delta \Gamma}{\delta \bar{\phi}_1} \right|_{\bar{\phi}_1 = \bar{\phi}_2 = \bar{\phi}} = \int d^4 y \sqrt{-g(y)}\, [\Gamma^{11}(x, y) + \Gamma^{12}(x, y)]\, \bar{\phi}(y) = 0, \tag{33}$$

which, taking into account Eqs. (31) and (32) can be expanded as

$$(-\Box_x + m^2)\, \bar{\phi}(x) + \int d^4 y \sqrt{-g(y)}\, \Sigma_{\mathrm{R}}(x, y)\, \bar{\phi}(y) = 0, \tag{34}$$

where $\Sigma_{\mathrm{R}}(x, y) = \Sigma^{11}(x, y) + \Sigma^{12}(x, y)$ is the retarded self-energy. The above equation of motion is real, because the retarded self-energy is a purely real quantity when expressed in configuration space (although it can develop an imaginary part in momentum space).

In flat spacetime the above equation is expressed as

$$(-\Box_x + m^2)\,\bar{\phi}(x) + \int d^4x'\;\Sigma_{\rm R}(x,x')\,\bar{\phi}(x') = 0, \qquad (35)$$

which, introducing the Fourier transform along the lines of Eq. (9), can be written as

$$p^2 + m^2 + \Sigma_{\rm R}\,(p;X) = 0, \qquad (36)$$

where we recall that $X = (x + x')/2$. Thus, in flat spacetime the self-energy and effective action methods lead to the same result provided we use a CTP approach in both situations and we neglect vertices with three external particles or more. Notice that the equivalence between both methods extends to arbitrary non-equilibrium situations, hence the X dependence in Eq. (36). In other words, Eq. (36) amounts to finding the poles of the retarded propagator even in a nonequilibrium situation, see Eq. (10).

In curved spacetime we cannot introduce the Fourier transform. However, we still can expand the right-hand side of Eq. (34) as follows:

$$(-\Box_x + m^2)\,\bar{\phi}(x) + \sum_n E^{\mu_1\cdots\mu_n}(x)\,\nabla_{\mu_1}\cdots\nabla_{\mu_n}\,\bar{\phi}(x) = 0. \qquad (37)$$

If the effective action is local, as in the case of Drummond and Hathrell, the above expansion can be truncated. We can introduce the geometric optics approximation: we suppose that the field is of the form $\bar{\phi} = A(x)\exp[iS(x)]$, with the phase S varying much more rapidly than A. Then, the leading contribution to the equations of motion can be written as

$$k^2 + m^2 + \sum_n i^n E^{\mu_1\cdots\mu_n}(x) k_{\mu_1}\cdots k_{\mu_n} = 0, \qquad (38)$$

where $k_\mu(x) = \nabla_\mu S(x)$.

We thus see that the geometric optics approximation essentially corresponds to the Fourier transform in flat spacetime. An expression analogous to Eq. (38), but particularized to the case of a photon in QED, was obtained by Drummond and Hathrell. However they used a conventional in-our approach instead of a CTP approach. Let us analyze the differences. Within the in-out formalism, Eq. (34) would read:

$$(-\Box_x + m^2)\,\bar{\phi}(x) + \int d^4y\sqrt{-g(y)}\;\Sigma^{11}(x,y)\,\bar{\phi}(y) = 0. \qquad (39)$$

Recall that now $\bar{\phi}$ does not correspond in general to a true expectation value. In fact, in general $\Sigma^{11}(x,y)$ will be a complex quantity, so that we cannot guarantee that above equation of motion is real. The difference between the in-out and CTP approaches comes from the mixed term Σ^{12}. In flat spacetime the mixed components of the self-energy account for processes involving real particle production, and vanish whenever these processes do not happen. We can expect that a similar interpretation holds in curved spacetime. The cases studied by Drummond and Hathrell did not lead to any particle production, neither electromagnetically (because of the lack of phase space for real particle production in all e^2 diagrams) nor gravitationally (since the creation of particles with energies much higher than the inverse curvature radius is strongly suppressed within the adiabatic approach, see Birrell and Davies, 1982). Hence, it is likely that their result is not significatively modified by the consideration of the equations of motion of the true expectation values. Nevertheless, we believe that this point still deserves a more careful analysis. For instance, when considering fields with small mass, particle production in unavoidable. Therefore, in these cases, only Eq. (34) would lead to real equations of motion.

ACKNOWLEDGMENTS

We are grateful to E. Gunzig for the organization of the Peyresq Physics meetings which triggered the present work. We thank B. L. Hu and A. Roura for interesting discussions there. D. A. acknowledges support of a FI grant from the Generalitat de Catalunya. This work is also partically supported by MICYT Research Project No. FPA-2001-3598.

REFERENCES

Adler, S. L. (1971). Photon splitting and photon dispersion in a strong magnetic field. *Annals of Physics* **67**, 599–647.

Alfaro, J., Morales-Tecotl, H. A., and Urrutia, L. F. (2000). Quantum gravity corrections to neutrino propagation. *Physical Review Letters* **84**, 2318–2321 (gr-qc/9909079).

Alfaro, J., Morales-Tecotl, H. A., and Urrutia, L. F. (2002). Loop quantum gravity and light propagation. *Physical Review D* **65**, 103509 (hep-th/0108061).

Amelino-Camelia, G. and Piran, T. (2001a). Cosmic rays and TeV photons as probes of quantum properties of space-time. *Phys. Lett. B* **497**, 265–270 (hep-ph/0006210).

Amelino-Camelia, G. and Piran, T. (2001b). Planck-scale deformation of Lorentz symmetry as a solution to the UHECR and the TeV-gamma paradoxes. *Physical Reviews D*, **64**, 036005 (astro-ph/0008107).

Arteaga, D., Parentani, R., and Verdaguer, E. (2004). Propagation in a thermal graviton background. *Physical Review D*, **70**, 044019 (gr-qc/0311065).

Barton, G. (1990). Faster than c light between parallel mirrors: The Scharnhorst effect rederived. *Physics Letters B*, **237**, 559.

Birrell, N. D. and Davies, P. C. W. (1982). *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, England, UK.

Borgman, J. and Ford, L. H. (2003). The effects of stress tensor fluctuations upon focusing. (gr-qc/0307043).

Brout, R., Massar, S., Parentani, R., and Spindel, P. (1995). Hawking radiation without trans-Planckian frequences. *Physical Review D*, **52**, 4559–4568 (hep-th/9506121).

Bunch, T. S. and Parker, L. (1979). Feynman propagator in curved space-time: A momentum space representation. *Physical Review D*, **20**, 2499–2510.

Burgess, C. P., Cline, J., Filotas, E., Matias, J., and Moore, G. D. (2002). Loop generated bounds on changes to the graviton dispersion relation. *Journal of High Energy Physics*, **03**, 043 (hep-ph/0201082).

Calzetta, E. and Hu, B. L. (1987). Closed-time-path functional Formalism in curved spacetime: Application to cosmological back-reaction problems. *Physical Review D*, **35**, 495–509.

Campos, A. and Verdaguer, E. (1994). Semiclassical equations for weakly inhomogeneous cosmologies. *Physical Review D*, **49**, 1861–1880.

Campos, A. and Verdaguer, E. (1996). Stochastic semiclassical equations for weakly inhomegeneous cosmologies. *Physical Review D*, **53**, 1927–1937.

Carroll, S. M., Field, G. B. and Jackiw, R. (1990). Limits on a Lorentz and parity violating modification of electrodynamics. *Physical Review D*, **41**, 1231.

Carroll, S. M., Harvey, J. A., Kostelecký, V. A., Lane, C. D., and Okamoto, T. (2001). Noncommutative field theory and Lorentz violation. *Physical Review Letters*, **87**, 141601 (hep-th/0105082).

Chou, K.-C., Su, Z.-B., Hao, B.-L. and Yu, L. (1985). Equilibrium and nonequilibrium formalisms made unified. *Physics Reports*, **118**, 1–131.

Corley, S. and Jacobson, T. (1996). Hawking spectrum and high frequency dispersion. *Physical Review D*, **54**, 1568–1586 (hep-th/9601073).

Das, A. (1997). *Finite Temperature Field Theory*, World Scientific, Singapore.

Donoghue, J. F. (1994a). General relativity as an effective field theory: The leading quantum corrections. *Physical Review D*, **50**, 3874–3888.

Donoghue, J. F. (1994b). Leading quantum correction to the newtonian potential. *Physical Review Letters*, **72**, 2996–2999.

Donoghue, J. F., Holstein, B. R., and Robinett, R. W. (1985). Renormalization and radiative corrections at finite temperature. *Annals of Physics* (NY), **164**, 233.

Drummond, I. T. and Hathrell, S. J. (1980). QED vacuum polarization in a background gravitational field and its effect on the velocity of photons. *Physical Review D*, **22**, 343.

Helfer, A. D. (2003). Do black holes radiate? *Reports on Progress in Physics*, **66**, 943–1008 (gr-qc/0304042).

Hu, B. L. and Verdaguer, E. (2003). Stochastic gravity: A primer with applications. *Classical and Quantum Gravity*, **20**, R1–R42 (gr-qc/0211090).

Hu, B. L. and Verdaguer, E. (2004). Stochastic gravity: Theory and applications. *Living Reviews in Relativity*, **7**, 3 (gr-qc/0307032).

Jacobson, T. (1991). Black hole evaporation and ultrashort distances. *Physical Review D*, **44**, 1731–1739.

Jacobson, T. (1933). Black hole radiation in the presence of a short distance cutoff. *Physical Review D*, **48**, 728–741 (hep-th/9303103).

Jacobson, T. (1999). Trans-Planckian redshifts and the substance of the space-time river. *Progress on Theoretical Physics Supplement*, **136**, 1–17 (hep-th/0001085).

Jacobson, T., Liberati, S., and Mattingly, D. (2003a). A strong astrophysical constraint on the violation of special relativity by quantum gravity. *Nature*, **424**, 1019–1021.

Jacobson, T., Liberati, S., and Mattingly, D. (2003b). Threshold effects and Planck scale Lorentz violation: Combined constraints from high energy astrophysics. *Physical Review D*, **67**, 124011 (hep-ph/0209264).

Jacobson, T., Liberati, S., Mattingly, D., and Stecker, F. W. (2004). New limits on Planck scale Lorentz violation in QED. *Physical Review Letters*, **93**, 021101 (astro-ph/0309681).

Jordan, R. D. (1986). Effective field equations for expectation values. *Physical Review D*, **33**, 444–454.

Keldysh, L. V. (1965). Diagram technique for nonequilibrium processes. *Zh. Eksp. Teor. Fiz*, **47**, 1515 [*Sov. Phys. JEPT* **20**, 1018 (1965)].

Kifune, T. (1999). Invariance violation extends the cosmic ray horizon? *Astrophysical Journal*, **518**, L21–L24 (astro-ph/9904164).

Kostelecký, V. A. and Mewes, M. (2001). Cosmological constraints on Lorentz violation in electrodynamics. *Physics Review Letters*, **87**, 251304 (hep-ph/0111026).

Kostelecký, V. A. and Samuel, S. (1989). Spontaneous breaking of Lorentz symmetry in string theory. *Physical Review D*, **39**, 683.

Landsman, N. P. and van Weert, C. G. (1987). Real- and imaginary-time field theory at finite temperature and density. *Physics Reports*, **145**, 141–249.

Latorre, J. I., Pascual, P., and Tarrach, R. (1995). Speed of light in nontrivial vacua. *Nuclear Physics B*, **437**, 60–82 (hep-th/9408016).

le Bellac, M. (1996). Thermal Field Theory, Cambridge University Press, Cambridge, England, UK.

Martin, J. and Brandenberger, R. H. (2001). The trans-Planckian problem of inflationary cosmology. *Physical Review D*, **63**, 123501 (hep-th/0005209).

Martin, J. and Brandenberger, R. (2003). On the dependence of the spectra of fluctuations in inflationary cosmology on trans-Planckian physics. *Physical Review D*, **68**, 063513 (hep-th/0305161).

Martín, R. and Verdaguer, E. (2000). Stochastic semiclassical fluctuations in Minkowski spacetime. *Physical Review D*, **61**, 124024 (gr-qc/0001098).

Niemeyer, J. C. (2001). Inflation with a high frequency cutoff. *Physical Review D*, **63**, 123502 (astro-ph/0005533).

Niemeyer, J. C. and Parentani, R. (2001). Trans-Planckian dispersion and scale-invariance of inflationary perturbations. *Physical Review D*, **64**, 101301 (astro-ph/0101451).

Parentani, R. (2001). Quantum metric fluctuations and Hawking radiation. *Physical Review D*, **63**, 041503 (gr-qc/0009011).

Parentani, R. (2002). Beyond the semiclassical description of black hole evaporation. *International Journal of Theoretical Physics*, **41**, 2175–2200.

Peskin, M. E. and Schroeder, D. V. (1998). *An Introduction to Quantum Field Theory*, Addison-Wesley, Reading, MA.

Scharnhorst, K. (1990). On propagation of light in the vacuum between plates. *Physics Letters B*, **236**, 354–359.

Schwinger, J. S. (1961). Brownian motion of a quantum oscillator. *Journal of Mathematical Physics*, **2**, 407.

Shore, G. M. (2002a). Faster than light photons in gravitational fields. II: Dispersion and vacuum polarisation. *Nuclear Physics B*, **633**, 271–294 (gr-qc/0203034).

Shore, G. M. (2002b). A local effective action for photon gravity interactions. *Nuclear Physics B*, **646**, 281–300 (gr-qc/0205042).

Shore, G. M. (2003). Quantum gravitational optics. *Contemporary Physics*, **44**, 503–521 (gr-qc/0304059).

't Hooft, G. (1996). The scattering matrix approach for the quantum black hole: An overview. *International Journal of Modern Physics A*, **11**, 4623–4688 (gr-qc/9607022).

Takeda, M., *et al.* (1998). Extension of the cosmic-ray energy spectrum beyond the predicted Greisen–Zatsepin–Kuzmin cutoff. *Physical Review Letters*, **81**, 1163–1166 (astro-ph/9807193).

Tarrach, R. (1983). Thermal effects on the speed of light. *Physics Letters B*, **133**, 259.

Tomboulis, E. (1977). $1/N$ expansion and renormalization in quantum gravity. *Physics Letters B*, **70**, 361.

Unruch, W. G. (1995). Sonic analog of black holes and the effects of high frequencies on black hole evaporation. *Physical Review D*, **51**, 2827–2838.

Weinberg, S. (1995). *The Quantum Theory of Fields*, Vol. I. Foundations, Cambridge University Press, Cambridge.

Weldon, H. A. (1983). Simple rules for discontinuities in finite-temperature field theory. *Physical Review D*, **28**, 2007–2015.